BOWLAND MATHS
Assessing the Case Studies

Keeping the pizza hot
Assessing the learning

Case Study description

Pupils explore the real-world scenario of delivery of pizzas

Suitability National Curriculum Levels 6 to 8

Time The assessment activities can be completed within the times for the case study

Resources All the resources required are listed within the case study

Opportunities to assess Key Processes

The lesson phases are inter-related, so evidence of various Key Processes may be seen at any time. The following reflects evidence gathered during trials.

- **Representing**: during phases 1 and 2
- **Analysing**: during phases 1 and 2
- **Interpreting and evaluating:** during phases 1 and 2
- **Communicating and reflecting**: during phase 3.

In addition to assessment of Key Processes, there are opportunities to assess Range and Content (detail is included within the case study), as well as some of the personal, learning and thinking skills, particularly those for 'team working'.

Phase 1: Creating a mathematical model

Pupils review data, preferably generated by themselves, and compare cooling rates for pizzas. Then they create a mathematical model to fit.

Teacher guidance

Observe how well pupils:

- Obtain and interpret data
- Decide which types of graph are appropriate
- Vary values to find the best fit
- Link their findings to the real world scenario of pizza cooling

Questions to ask:

- *What equations did you try?*
- *How did you decide what to change and how to change it?*
- *What are the advantages/disadvantages of using a straight line? Or a quadratic?*
- *How well does … predict what will happen?*
- *What assumptions did you make?*

Assessment guidance: Progression in Key Processes

	Representing	**Analysing**	**Interpreting and evaluating**
PROGRESSION	Understands the link between graphical representation and the pizza cooling	Improves the fit of an equation to the data	Recognises that a straight line implies the pizza cools at a constant rate
	Compares linear equations to find the best fit	As above and shows understanding of the effect of varying intercept and gradient	As above, and uses their knowledge of intercept and/or gradient to apply to the real life situation Pupil pair A
	Tries out and compares different representations, eg quadratic curves Pupil pair A	Shows understanding of the effect of varying values for quadratics or exponentials Pupil pair A	Explains why both linear and quadratics are not appropriate representations,
	Tries out and compares different representations, eg quadratic and exponential curves Pupil pair B; Pupil pair D (p8)	Gives an exponential equation that fits the data reasonably well Pupil pair B; Pupil pair D (p8)	Explains why an exponential function is the most appropriate function for the given context Pupil pair B; Pupil pair D (p8)

Sample response: Pupil pair A

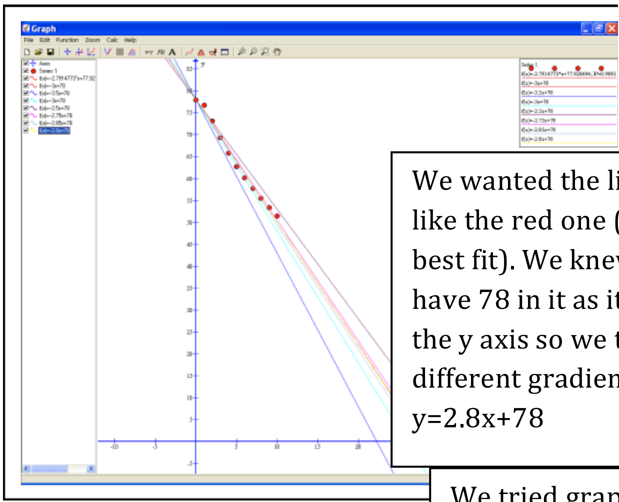
Comments

The pupils vary values within linear equations systematically, recognising the need to keep the y-intercept constant. Their trials for quadratics show an increasing awareness of the effect of varying values, but they make little progress with the exponential function.

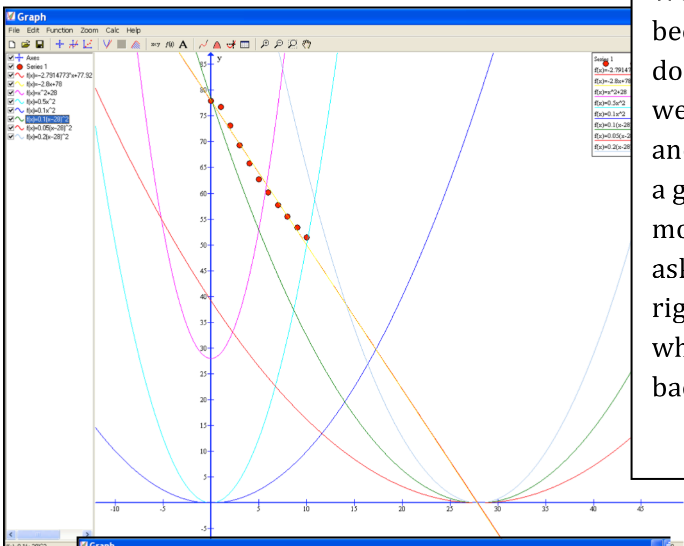
Probing questions and feedback

- *How did you know to try a negative gradient when you were working with straight lines?*
- *What was the effect of the changes you made when you were working with quadratics? Can you predict what would happen if …. ?*

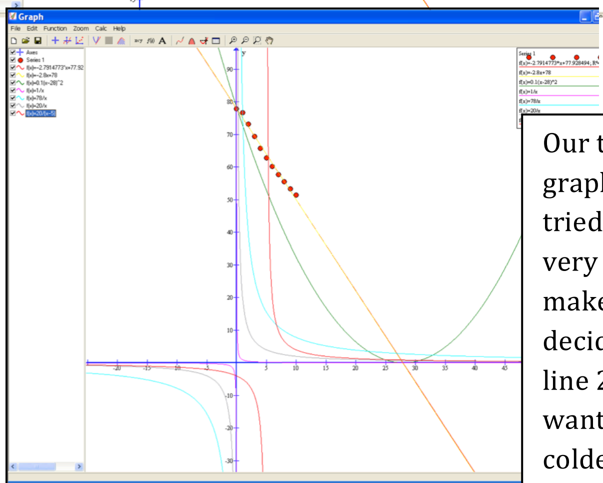
The pupils would benefit from further discussion to find answers to their questions, ie what would the up slope of the quadratic mean, and so why might that not be a good fit?



We wanted the line to be like the red one (the line of best fit). We knew it would have 78 in it as it is 78 on the y axis so we tried different gradients and got y=2.8x+78



We tried graphs of x^2 because they curve down. We got one that went through (0,78) and (28,0) but it wasn't a good fit so we tried more. Our teacher asked if the x^2 was right and what it meant when the graph went back up.



Our teacher said to try graphs like y = 1/x but we tried and they were not very good and we couldn't make them fit well so we decided the best fit was the line 2.8x+78 because we want to deliver before it is colder than 62 degrees so this one fits ok in the

Sample response: Pupil pair B

Keeping The Pizza Hot

In our group we were trying to solve the problem of where to place a new pizza shop. Our criteria for the shop was that it had to deliver to a large area whilst still keeping the pizzas warm.

We carried out two separate experiments which were:

- Leaving a pizza on plate
- A pizza wrapped in foil

In each experiment we measured the temperature of the pizza every minute using a thermometer.

Pizza wrapped in foil

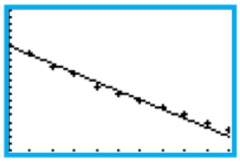
We carried out the experiment as above and these are the results we ended up with. We have also included our prediction and how we predicted this:

Time (minutes)	Temperature (°C)	Prediction	How did you make your prediction?
0	76		
1	74	68	Guess. Thought it would go down by 2°C
2	72	72	As Above
3	71	70	As Above
4	69	68	As Above
5	68	66	As Above (1°C)
6	67	66	As Above
7	66	66	As Above
8	65	65	Heat has been going down 1°C at a time.
9	64	64	As Above
10	63	63	As Above

First, we tried the equation **$y = -1.5x+70$** but it looked a bit too steep and was too low down. Then we tried **$y = -1x+70$**, but it was still too low and now too shallow.

After that we tried **$y = -1.75x + 74$**, but not it was just too steep and still too low. We then tried **$y = -1.55x + 76$**, this line was better but a bit too steep. We realised the line was a bit too high so, we decreased the intercept to 75. We also noticed the gradient was still too steep, so we changed this to **-1.3.**

Our final line was **$y = -1.3x + 75$**, which fit well for the data we collected. However the pizza would remain at room temperature and not keep cooling, so a straight line would not fit after a longer time but we did not find time to fit a curve.



Pizza on a plate

In our experiment we were trying to find out how fast the temperature decreased in a pizza left on a plate. We measured the temperature every minute.

Time (Minutes)	Temperature °c
0	70
1	70
2	68
3	66
4	64
5	62
6	60
7	58
8	57
9	56
10	55

We then drew a scatter graph which showed a gradual decrease of temperature. Then using a graphical calculator we began trying to fit a straight line through this by using the equation **$y = mx + c$.**

We began with the graph **$y = -x$** as the graph had negative correlation. This line did not work so we then added an intercept of 75, giving us the line **$y = x + 75$.** This graph was too shallow and high up. We then decreased the intercept to 70. This graph was closer but still too shallow. We then added in the gradient of -2 (**$y = -2x + 70$**).

This became too steep, so we slightly decreased the gradient to -1.5. This was a lot closer but too low. We then changed the intercept back to 75 (**$y = -1.5x + 75$**). This was too high so we decreased the intercept to 71. We then increased the gradient to -1.65. This was near perfect so we increased the gradient slightly to -1.68.

$Y = -1.68x + 71$ – This was the best we found.

This graph has a steeper gradient than the pizza in foil, which shows that the pizza on a plate cools quicker. The intercept is slightly lower because it has already cooled more than the pizza in foil when we took the first measurement. This is because we didn't measure as soon as the pizza came out of the oven because it took a couple of minutes to get to maths from the food tech room.

We then used this equation to work out the temperature at 60 minutes.

$60 \text{ X } -1.68 + 71 = -298$

This could not be true as the pizza would remain at room temperature. This means I cannot extrapolate the data. We then concluded the graph must be a curve. To fit this curve we began with the equation **$y = 1/x$.** We thought this graph has the right sort of shape because a quadratic graph would get hotter again and a cubic is the wrong shape. This was nowhere near close enough. We next tried adding numbers to x in increments of 10, until we reached **$y = 1/(x + 60)$.**

This was a lot closer than any prior attempts, so we added on a further 5. This gave us the line **$y = 1/(x + 65)$.** Although this was high enough it was too steep. We then tried **$y = 15/(x + 65)$.** This then became too high so we took. This was a near perfect fit, so we increased the number divided by x to 16 and then 18.

This was a near perfect fit.

We then tried a variety of methods including increasing the gradient, multiplying x and trial and improvement until we found a close enough fit.

We also realised that we would have to add 20 to the equation so the graph levels out at 20 degrees because the pizza will stay at room temperature.

Comments

The pupils gathered data. They varied values within linear equations, comparing and interpreting them. They chose to use an exponential function, using mathematical insight when varying values.

Probing questions and feedback

- *If you add a constant to the equation, what happens to its graph?*
- *Does it matter if the equation is linear, or quadratic, or exponential?*

This pair would benefit from working on other activities that help them to link their knowledge of equations to the real world.

Phase 2: Deciding where deliveries can be made

Pupils choose the packaging and use their model to find for how many minutes the pizza stays ‘hot’. They then establish where, within an area, they can deliver in that time.

Teacher guidance

Observe how well pupils:

- Use their model to find the time available
- Find the maximum distance that can be travelled within this time
- Link their findings to the real world scenario of pizza delivery

Questions to ask:

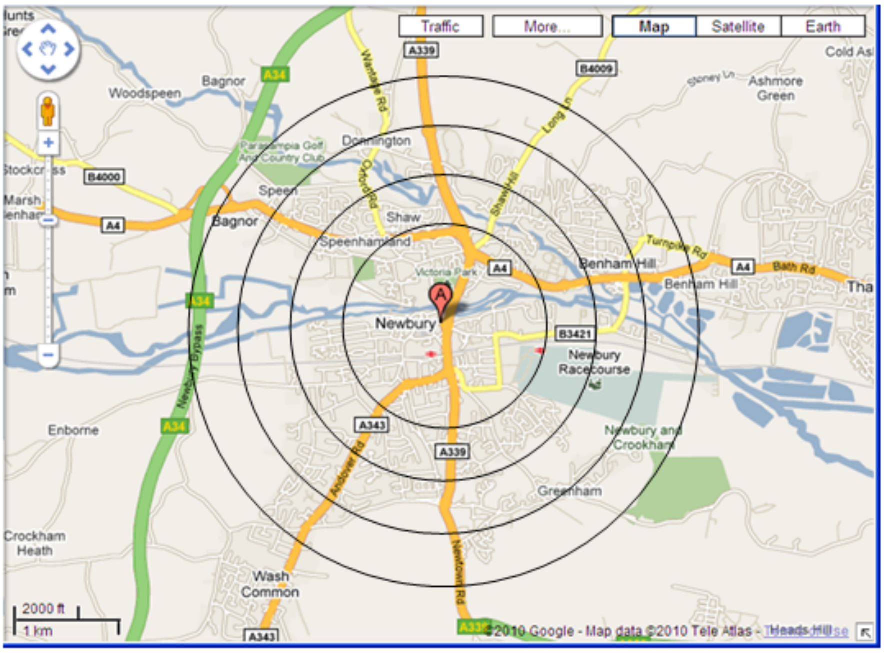
- *How confident are you that the maximum time for delivery is about right?*
- *How did you decide where you can and can’t deliver?*
- *What assumptions have you made?*
- *What else do you need to think about to improve your estimate of the delivery area?*

Assessment guidance: Progression in Key Processes

	Representing	**Analysing**	**Interpreting and evaluating**
PROGRESSION	Needs teacher support to select appropriate tools	Shows understanding of scale Pupil C	Interprets findings within the context, eg shows distances on a map Pupil C
	Selects appropriate tools for some of the steps Pupil C	Shows understanding of speed and scale	Uses their previous work to determine the time available
	Selects appropriate tools for all of the steps Pupil pair D (p8)	Uses speed and scale efficiently and effectively	Gives a clear solution that takes the major factors into account Pupil pair D (p8)
	Selects appropriate tools for all the steps and shows insight into the problem, eg by varying speed according to type of road	Uses speed, scale and locus efficiently and effectively Pupil pair D (p8)	Recognises that their model is a simplification, ie that other factors impact

Sample response: Pupil C

How far you can deliver depends on time of day because Newbury gets clogged up with traffic. The circles show how far you can get at different times. The biggest circle is 5km away; then it is 4km, 3km and 2km. If you listen to local radio you will get an idea of how much traffic there is and then you can decide if you can get there in time or not.



Comments

Although pupil C recognised the complexity of the problem, her solution lacked detail.

Probing questions and feedback

- *What other information would the pizza shop owner need to decide how far she could get?*
- *If you were travelling at 30mph, say, where could you get to before the pizza cools? How do you know? If you were travelling at 60mph, would you go twice as far? What about 45mph? Or 40mph? Or … ?*

Pupil C would benefit from reviewing work done by other pupils. This should support her in identifying and explaining key information.

Phase 3: Producing a report

Pupils produce a report for a given audience, eg the pizza shop owner.

Teacher guidance

Observe how well pupils:

- Summarise their findings, giving key information
- Tailor their report to the relevant audience

Questions to ask:

- *When you listen to (or see) other groups' presentations (or reports), what will you be looking for and why?*
- *What is the difference between a report for a scientific journal, say, and a report for the pizza shop owner?*
- *Is there anything else the person reading your report needs to know? Is there any detail that they are unlikely to want to know?*

Assessment guidance: Progression in Key Processes

This phase may also give evidence of representing, analysing, interpreting and evaluating. The tables shown above can be used to evaluate progression in these areas.

PROGRESSION	**Communicating and reflecting**
↓	Creates a simple report that explains what they have done and why; gives simple feedback to others
↓	Creates a report that identifies clear conclusions; gives helpful feedback to others Pupil pair D
↓	Communicates effectively; gives effective feedback and reflects on own approach
↓	Communicates effectively and concisely; gives insightful feedback and reflects on a range of approaches Pupil pair E

Sample response: Pupil pair D

Math Pizza Experiment

In our maths lessons we have been helping a pizza company work out how far it can deliver pizzas before they go cold. We had the task of working out the area in which they could deliver too deciding a suitable place for the shop and how long the riders had to deliver the pizza.

During the course of the first lesson we had two pizzas. One we left on the table to cool normally and the other we made a pizza box for. We measured the temperature at different point and recorded them.

Here are the results for our pizza in a box.

Time (T) (minutes)	Temp (t) (deg.C)	Prediction	How did you make your prediction?
0	73	75	It's very hot straight out of the oven
1	73	70	I thought it would cool rapidly in the 1st minutes
2	73	67	
3	71	67	
4	69	69	
5	68	67	It seems to be cooling a degree or 2 every minute
6	66	67	
7	65	64	
8	64	63	
9	63	62	
10	61	60	
30		50	I thought it would cool slower as time goes on
120		40	
24 hours		21	I don't know for definite but it's roughly room temperature

In the second lesson we took our results and put them into the graphic calculators.

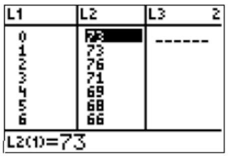
Comments

The pupils gave a detailed account of their work but did not summarise it for their chosen audience (a scientific magazine). However, their feedback showed a growing awareness of the purpose of their report.

Probing questions and feedback

- *What are the main differences between these articles (in a scientific magazine) and your report?*
- *If you were sending your report to be published, what would you change and why?*

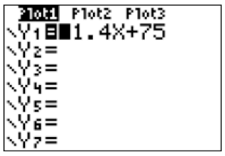
The pupils would benefit from group discussion about how to create a report that includes key findings and methods, yet is concise.

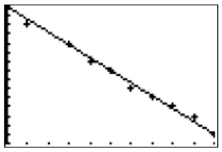


I noticed afterwards that I made a mistake and put 76 instead of 73 for the 3rd reading (2 minutes).

We fitted a straight line to our data. I thought the y-intercept would be 75, not 73, because the pizza would be a bit hotter when it first came out the oven because we didn't measure it straight away. Also this made the line fit the rest of the points better.

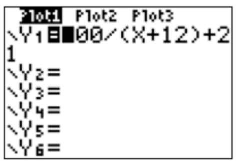
If I assume the first reading should be 75, the pizza cooled from 75 to 61 in ten minutes. This is 14 degrees. This means the gradient of the graph should be -1.4 because the pizza needs to cool 1.4 degrees every minute.

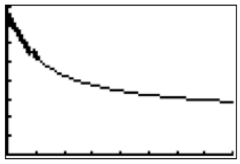




In the next lesson we needed to change our straight line as we found a problem with it, so we now used a curve. The problem with a straight line was that if the line was straight it would go down into minus temperatures instead of levelling off at room temperature as it would do. With a straight line we can only interpolate between roughly 0 and 10 minutes and could not extrapolate. A curve would suit the graph more.

We thought a reciprocal graph would be the best shape. We know we had to add 21 to the equation to move the graph up, so the pizza cools to 21 degrees which is about room temperature. We tried lots of different equations but this seemed to fit best.





We watched a video of a boy taste testing the pizza. He thought the pizza was too cold at 62 degrees. This means it would take between 9 and 10 minutes to get too cold. To simplify our calculations we will approximate the time to 10 minutes.

We decided that the pizza shop was in the precinct. The time the pizza took to cool was about 10 minutes. This is 10 ÷ 60 = 0.16 recurring hours, so that was the time we had to deliver the pizza. The speed limit around the precinct is 30mph.

Speed = distance ÷ time

So distance = speed x time
= 30 x 0.1666
= 5 miles

Or to find how many miles in 30 minutes you would half the miles because you halved the minutes from 60 to 30. This was 15 miles for 30 minutes. We needed to get to 10 minutes so we divided the minutes and miles again by 3. This gave us 5 miles in 10 minutes.

This tells us the radius of the circle (the circle is how far you can travel) is 5 miles. Then you just scale the circle to the map scale and draw it on.

We found a map with a scale of 1cm on the map representing 200m.

5 miles is about 8km = 8000m so this will be 8000 ÷ 200 = 40cm so we would need to draw a circle with a radius of 40 cm with the centre of the circle at the precinct. This was too big to fit on the map we found.

Sample response: Pupil pair E

Pizza delivery by YCA pizza

Our packaging. We recommend foil. Experiments showed that Jiffy bags kept the temperature better but they will get dirty and it is not hygienic. You could use foil and then jiffy bags but that is expensive and it uses too much stuff for the environment.

How much time you have to deliver. We created an exponential graph which estimates 8 minutes before it is at 62° but if the traffic is bad the pizza is cold so we recommend 6 minutes so that you will have some time to spare.

How far you can go in 6 minutes. We think 30mph is realistic because it is a town and there is a speed limit and 6 minutes is one tenth of an hour so you can deliver 3 miles.

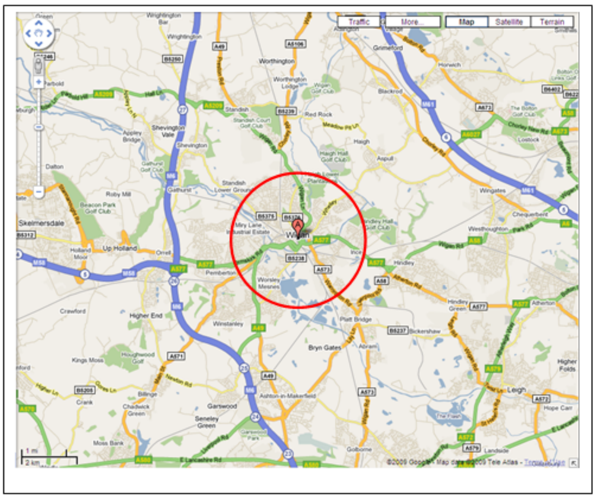
Where can you go?

We know that 1mile=1.6km so 3miles=4.8km so we used the scale.

——— is 2km

—————— is 4.8km

So you can deliver anywhere in the red circle but if someone lives just outside you should look to see if it was a fast road so you could get there quicker.



Comments

The pupils summarised their findings when writing a report for the pizza shop owner. They identified the key features and offerred clear and concise conclusions. They also gave oral feedback which was helpful to others and identified features that could be improved in order to communicate more effectively for the given audience.

Probing questions and feedback

- *If you were writing a report for a scientific journal, what would you change and why?*

The pupils would benefit from reviewing different types of report and discussing the key features of each.